

Effect of the Charged Higgs Bosons in the Radiative Leptonic Decays of B^- and D^- Mesons

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Abstract

In this work, we study the radiative leptonic decays of B^- and D^- mesons in the standard model and the two-Higgs-doublet-model type II. The results are obtained using the factorization procedure, and the contribution of the order $O(\Lambda_{\text{QCD}}/m_Q)$ is included. The numerical results are calculated using the wave-function obtained in relativistic potential model. As a result, the decay mode $B \rightarrow \gamma \tau \nu_\tau$ is found to be sensitive to the effect of the charged Higgs boson. Using the constraint on the free parameters of the two Higgs doublet model given in previous works, we find the contribution of the charged Higgs boson in the decay mode $B \rightarrow \gamma \tau \nu_\tau$ can be as large as about 13%.

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I. INTRODUCTION

A resonance at 126 GeV is discovered in both the ATLAS [1] and CMS experiments [2], which is consistent with the Standard Model (SM) Higgs boson [3]. This is one of the great experimental achievement in recent years. Despite the great success of SM, there are still problems implying new physics beyond SM. One of the simplest extensions is the two Higgs doublet model (2HDM), with a second Higgs doublet introduced by Ref. [4] (see also [5]). The Higgs potential and the Yukawa Lagrangian corresponding to the 2HDM is not unique. The one with a CP-invariant potential, and with the Yukawa Lagrangian such that one of the Higgs doublet couples to the down sector of fermions while the other couples to the up, which is so called 2HDM-Type-II [6]. Such a Lagrangian can avoid tree-level flavor changing neutral currents (FCNCs) [7]. There are 2 Vacuum Expectation Values (VEV), denoted as v_1 , and v_2 in 2HDM, and 5 Higgs bosons denoted as h , A , H^0 and H^\pm . By studying the phenomena induced by 2HDM, and comparing the results with the experimental data, one can detect the signal of the existence of new physics, and fix the free parameters in the model.

The radiative leptonic decay of the heavy pseudoscalar meson with a massive lepton provides a good opportunity to study the 2HDM. This process can be mediated both by W bosons and H^\pm bosons, however, the contribution of the charged Higgs bosons is barely investigated before. The contribution of 2HDM is usually small. To probe the signal, sufficient precision of the numerical results in the SM is required. In this decay mode, the strong interaction is involved only in the initial hadronic state. As a result, this decay mode has been investigated using the factorization approach [8, 9], and results has been calculated up to the order $O(\alpha_s \Lambda_{\text{QCD}}/m_Q)$ in the SM [9]. Apart from that, with a massive lepton in the final state, the results are expected to be sensitive to the charged Higgs bosons.

There are two free parameters in 2HDM associated with this decay mode, $\tan \beta \equiv v_2/v_1$, and M_{H^\pm} . The branching ratios can be expressed as the function of $R \equiv \tan \beta/M_{H^\pm}$. Using the constraints on those parameters [10–13], the branching ratios including the contribution of the charged Higgs bosons can be obtained. The results are shown in Table IV. We find that, the decay mode $B \rightarrow \gamma \tau \nu_\tau$ is very sensitive to the charged Higgs bosons, which makes it a good decay mode to probe the signal of 2HDM. On the other hand, the decay mode $D \rightarrow \gamma \mu \nu_\mu$ is barely affected by 2HDM, and should be excluded in the search of the charged

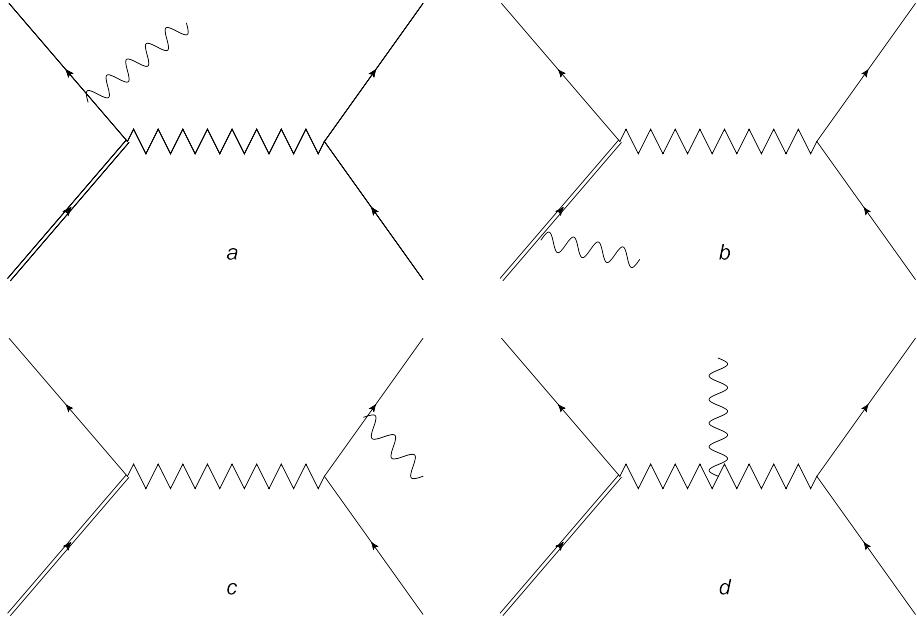


FIG. 1: tree level amplitudes in SM, the double line represents the heavy quark propagator but not the HQET propagator.

Higgs bosons.

This paper is organized as follows. Sec. II is a briefly review of the factorization in the SM. The contribution of the 2HDM is discussed in Sec. III. The numerical results and analysis are contained in Sec. IV. Sec. V is a summary.

II. FACTORIZATION IN STANDARD MODEL

The heavy pseudoscalar meson is constituted with a quark and an anti-quark, and one of the quarks is a heavy quark. The Feynman diagrams of the radiative leptonic decay of B meson at tree level can be shown as Fig. 1. The contribution of Fig. 1. d is suppressed by a factor of $1/M_w^2$, and can be neglected. The amplitudes of Fig. 1. a, b and c can be written as

$$\begin{aligned}
 \mathcal{A}_a^{(0)} &= \frac{-ie_q G_F V_{Qq}}{\sqrt{2}} \bar{q}(p_{\bar{q}}) \not{\epsilon}_{\gamma}^* \frac{\not{p}_{\gamma} - \not{p}_q}{2p_{\gamma} \cdot p_{\bar{q}}} P_L^\mu Q(p_Q) (\bar{l} P_{L\mu} \nu) \\
 \mathcal{A}_b^{(0)} &= \frac{-ie_Q G_F V_{Qq}}{\sqrt{2}} \bar{q}(p_{\bar{q}}) P_L^\mu \frac{\not{p}_Q - \not{p}_{\gamma} + m_Q}{2p_Q \cdot p_{\gamma}} \not{\epsilon}_{\gamma}^* Q(p_Q) (\bar{l} P_{L\mu} \nu) \\
 \mathcal{A}_c^{(0)} &= \frac{-e G_F V_{Qq}}{\sqrt{2}} \bar{q}(p_{\bar{q}}) P_L^\mu Q(p_Q) \left(\bar{l} \not{\epsilon}_{\gamma}^* \frac{i(\not{p}_{\gamma} + \not{p}_l + m_l)}{2(p_{\gamma} \cdot p_l)} P_{L\mu} \nu \right)
 \end{aligned} \tag{1}$$

where $p_{\bar{q}}$ and p_Q are the momenta of the anti-quark \bar{q} and quark Q , p_γ , p_l and p_ν are the momenta of photon, lepton and neutrino, ε_γ denotes the polarization vector of photon, and $P_L^\mu \equiv \gamma^\mu(1 - \gamma_5)$. The idea of factorization is to absorb the infrared (IR) behaviour into the wave-function, the matrix element can be written as the convolution of wave-function and hard kernel [8, 14]

$$F = \int dk \Phi(k) T_{\text{hard}}(k) \quad (2)$$

For simplicity, we denote

$$x = m_Q^2, \quad y = 2k_Q \cdot p_\gamma, \quad z = 2p_\gamma \cdot k_{\bar{q}}, \quad w = 2k_Q \cdot k_{\bar{q}} \quad (3)$$

Using the wave-function defined in coordinate space

$$\Phi_{\alpha\beta}(x, y) = < 0 | \bar{q}_\alpha(x) [x, y] Q_\beta(y) | \bar{q}^S(p_{\bar{q}}), Q^s(p_Q) > \quad (4)$$

where $[x, y]$ is the Wilson Line which can be written as [15]

$$[x, y] = \exp \left[ig_s \int_y^x d^4 z z_\mu A^\mu(z) \right] = \sum_n \frac{(ig_s)^n}{n!} \prod_i^n \int_y^x d^4 z_i z_{i\mu} A^\mu(z_i) \quad (5)$$

it has been proved that [9], the matrix element up to the order $O(\alpha_s \Lambda_{\text{QCD}}/m_Q)$ at one-loop can be factorized as

$$F^\mu(\mu) = \sum_n \int d^4 k_Q \int d^4 k_{\bar{q}} \Phi(k_Q, k_{\bar{q}}) C_n(k_Q, k_{\bar{q}}, \mu) T_n(k_Q, k_{\bar{q}}) \quad (6)$$

with

$$\begin{aligned} T_1 &= -e_q \frac{\not{\epsilon}_\gamma \not{p}_\gamma}{2p_\gamma \cdot k_{\bar{q}}} P_L^\mu, \quad T_2 = e_q \frac{2\varepsilon_\gamma^* \cdot k_q}{2p_\gamma \cdot k_{\bar{q}}} P_L^\mu, \quad T_3 = e_Q P_L^\mu \frac{\not{p}_\gamma \not{\epsilon}_\gamma^*}{2k_Q \cdot p_\gamma} \\ T_4 &= e_Q P_L^\mu \frac{m_Q \not{\epsilon}_\gamma^*}{2k_Q \cdot p_\gamma}, \quad T_5 = \frac{-e \not{p}_\gamma \not{\epsilon}_\gamma^*}{2p_\gamma \cdot p_l} P_L^\mu + e P_L^\mu \left(\frac{\varepsilon \cdot p_P}{p_\gamma \cdot p_P} - \frac{\varepsilon \cdot p_l}{p_\gamma \cdot p_l} \right) \end{aligned} \quad (7)$$

except for $C_1 T_1$, all the other products are contribution of order $O(\Lambda_{\text{QCD}}/m_Q)$, for clarity, we define $C_1 = C_1^0 + C_1^1$, with C_1^m represents order $O((\Lambda_{\text{QCD}}/m_Q)^m)$ contribution, the coefficients are

$$\begin{aligned} C_1^0 &= \left\{ 1 + \frac{\alpha_s(m_Q) C_F}{4\pi} \left(-2 \text{Li}_2 \left(1 - \frac{y}{x} \right) - 2 \log^2 \frac{y}{x} - \frac{y}{x-y} \log \frac{y}{x} + 2 \log \frac{y}{x} - 6 - \frac{\pi^2}{12} \right) \right\} \\ &\times \exp \left(\frac{\alpha_s(m_Q) C_F}{4\pi} \left(-4 \log^2 \frac{\mu}{m_Q} + 4 \log \frac{y}{x} \log \frac{\mu}{m_Q} - 6 \log \frac{\mu}{m_Q} \right) \right) \\ &\times \left(1 + \frac{\alpha_s(\mu) C_F}{4\pi} \left(\log^2 \frac{\mu^2}{z} - 3 - \frac{\pi^2}{4} \right) \right) \end{aligned} \quad (8)$$

$$\begin{aligned}
C_1^1 = & \frac{w}{(x-y)^2 y} \left(x^2 \left(-4 \log \frac{xz}{y^2} - 2 \right) + xy \left(5 \log \frac{x}{y} + 8 \log \frac{z}{y} + 5 \right) + y^2 \left(2 \log \frac{y}{x} + 4 \log \frac{y}{z} - 3 \right) \right) \\
& + \frac{x^3 z}{(x-y)^2 y^2} \left(-3y_1 + 8 \log \frac{xz}{y^2} - 4 \right) + \frac{x^2 z}{(x-y)^2 y} \left(7y_1 + 13 \log \frac{y}{x} + 19 \log \frac{y}{z} + 14 \right) \\
& + \frac{xz}{(x-y)^2} \left(-5y_1 + 4 \log \frac{x}{y} + 14 \log \frac{z}{y} - 17 \right) + \frac{yz}{(x-y)^2} \left(y_1 - 3 \log \frac{z}{y} + 2 \log \frac{x}{y} + 7 \right) \\
& - \frac{w}{2(x-y)} \log \frac{x}{y} + \left(\frac{2w}{y} - \frac{4xz}{y^2} \right) \left(\log \frac{xz}{y^2} - \frac{1}{2}y_1 \right) + \left(\frac{2w}{y} - \frac{4xz}{y^2} \right) \left(\log \frac{x}{\mu^2} \log \frac{y}{z} - 2 \log \frac{y}{z} - \log \frac{x}{\mu^2} + 4 \right) \\
& - \left(2 - \frac{\pi^2}{3} - \log^2 \frac{y}{z} + \log \frac{y}{z} \log \frac{x}{\mu^2} + \log \frac{x}{\mu^2} \right) \frac{2z}{y}
\end{aligned} \tag{9}$$

$$\begin{aligned}
C_2 = & 1 + \frac{\alpha_s C_F}{4\pi} \left(-\log \frac{y}{\mu^2} + y_1 - 2 \log \frac{xz}{y^2} + \log \frac{x}{y} + \frac{xz}{yw} \left(\log \frac{xz}{y^2} - \frac{1}{2}y_1 \right) \right. \\
& \left. - \frac{4zx}{yw} \left(\log \frac{x}{\mu^2} \log \frac{y}{z} - 2 \log \frac{y}{z} - \log \frac{x}{\mu^2} + 4 \right) \right) \\
C_3 = & 1 + \frac{\alpha_s C_F}{4\pi} \left(\left(\log \frac{x}{\mu^2} - 4 - \frac{x}{y} \left(-\text{Li}_2(1 - \frac{y}{x}) + \frac{\pi^2}{3} \right) - \frac{6x-y}{x-y} \log \frac{x}{y} \right) \right. \\
& - 1 + \log \frac{x}{\mu^2} - \frac{x}{x-y} + \frac{y(2x-y)}{(x-y)^2} \log \frac{x}{y} + \frac{2x}{y} (-5 + 3 \log \frac{x}{\mu^2} + \frac{x}{x-y} + \frac{(2x-3y)y}{(x-y)^2} \log \frac{x}{y}) \\
& \left. - \log \frac{y}{\mu^2} - 2 + \frac{x}{y-x} + \frac{x^2}{(x-y)^2} \log \frac{x}{y} + \frac{8w}{w-z} \log \frac{y}{w-z} \right)
\end{aligned} \tag{11}$$

$$\begin{aligned}
C_4 = & \frac{\alpha_s C_F}{4\pi} \left(2 \left(\frac{2xz^2}{yw} - 2z \right) \frac{4w}{w-z} \log \frac{w-z}{y} \right. \\
& \left. - \frac{2y}{x-y} \log \frac{x}{y} + \left(-5 + 3 \log \frac{x}{\mu^2} + \frac{x}{x-y} + \frac{y(2x-3y)}{(x-y)^2} \log \frac{x}{y} \right) \right)
\end{aligned} \tag{12}$$

where y_1 is defined as:

$$y_1 = -\pi^2 + 2\text{Li}_2 \left(1 - \frac{x}{y} \right) - \log^2 \frac{xz}{y^2} + \log^2 \frac{x}{y} \tag{13}$$

In the results given above, the large logarithms at order $O((\Lambda_{\text{QCD}}/m_Q)^0)$ is already resummed. The $C_5 T_5$ term is the contribution from \mathcal{A}_c , and it dose not have 1-loop hard scattering kernel, so we have

$$C_5 = 1 \tag{14}$$

The numerical result can be obtained by using the wave-function with the one obtained

in Ref. [16]

$$\begin{aligned}\Phi(k_q, k_Q) &= \frac{1}{\sqrt{3}} \int d^3k \Psi(k) \frac{1}{\sqrt{2}} M |0\rangle \\ &\times \delta^3(\vec{k}_q + \vec{k}) \delta^3(\vec{k}_Q - \vec{k}) \delta(k_{q0} - \sqrt{k^2 + m_q^2}) \delta(k_{Q0} - \sqrt{k^2 + m_Q^2})\end{aligned}\quad (15)$$

with

$$M = \sum_i b_Q^{i+}(\vec{k}, \uparrow) d_q^{i+}(-\vec{k}, \downarrow) - b_Q^{i+}(\vec{k}, \downarrow) d_q^{i+}(-\vec{k}, \uparrow), \quad \Psi(\vec{k}) = 4\pi \sqrt{m_P \lambda_P^3} e^{-\lambda_P |\vec{k}|} \quad (16)$$

and the result is

$$\begin{aligned}<\gamma | \bar{q}\Gamma^\mu Q | P> &= \epsilon_{\mu\nu\rho\sigma} \varepsilon^\nu p_P^\rho p_\gamma^\sigma (F_V + F_{c1}) \\ &+ i (\varepsilon^\mu p_P \cdot p_\gamma - p_\gamma^\mu \varepsilon \cdot p_P) (F_A + F_{c1}) + F_{c2} p_P^\mu\end{aligned}\quad (17)$$

with

$$\begin{aligned}F_V &= \frac{1}{(2\pi)^3} \frac{3}{\sqrt{6}} \int d^3k \Psi(k) \frac{1}{2\sqrt{p_{q0}p_{Q0}(p_{q0} + m_q)(p_{Q0} + m_Q)}} \frac{1}{m_P E_\gamma} \\ &\times (2e_q C_1 m_Q - C_1^0 p_{q0} e_q + e_Q 2p_{q0} C_3) \\ F_A &= \frac{1}{(2\pi)^3} \frac{3}{\sqrt{6}} \int d^3k \Psi(k) \frac{1}{2\sqrt{p_{q0}p_{Q0}(p_{q0} + m_q)(p_{Q0} + m_Q)}} \frac{1}{m_P E_\gamma} \\ &\times \left(2e_q C_1 m_Q - C_1^0 p_{q0} e_q - e_q \frac{2p_{q0} m_Q}{E_\gamma} C_2 - e_Q 2p_{q0} C_3 + e_Q \frac{2p_{q0} m_Q}{E_\gamma} C_4 \right)\end{aligned}\quad (18)$$

where F_{c1} and F_{c2} come from the contribution of \mathcal{A}_c . Using

$$f_P p_P^\mu = i <0 | \bar{q}\gamma^\mu \gamma_5 Q | P> = -i <0 | \bar{q}\gamma^\mu (1 - \gamma_5) Q | P> \quad (19)$$

we find

$$F_{c1} = \frac{-ef_P}{2p_\gamma \cdot p_l}, \quad F_{c2} = ie f_P p_P^\mu \left(\frac{\varepsilon \cdot p_P}{p_\gamma \cdot p_P} - \frac{\varepsilon \cdot p_l}{p_\gamma \cdot p_l} \right) \quad (20)$$

and the decay amplitude can be written as

$$\mathcal{A}_{SM} = \frac{eG_f V_{Qq}}{\sqrt{2}} <\gamma | \bar{q}\Gamma^\mu Q | P> (\bar{l}P_L^\mu \nu) \quad (21)$$

III. 2HDM CONTRIBUTION

The Feynman diagrams at tree level of the radiative leptonic decay in 2HDM are shown in Fig. 2. The contribution of Fig. 2. d is suppressed by Higgs propagator and neglected, the decay amplitudes of the others can be written as

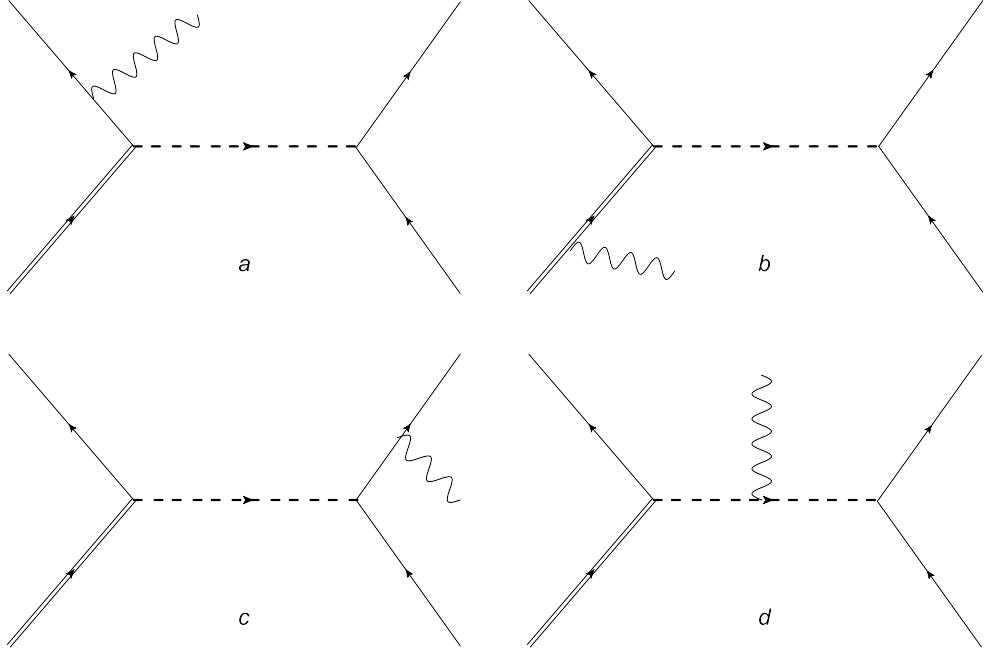


FIG. 2: Tree level amplitudes of charged Higgs bosons. The double line represents the heavy quark, and the dashed line, the charged Higgs.

$$\begin{aligned}
\mathcal{A}_a^{\text{2HDM}} &= \frac{i e_q g^2 V_{Qq} m_Q m_l \tan^2 \beta}{8 M_W^2 M_{H^+}^2} \bar{q}(p_{\bar{q}}) \not{f}_{\gamma}^* \frac{\not{p}_{\gamma} - \not{p}_{\bar{q}}}{(p_{\gamma} - p_{\bar{q}})^2 - m_q^2} (1 + \gamma_5) Q(p_Q) (\bar{l}(1 - \gamma_5) \nu) \\
\mathcal{A}_b^{\text{2HDM}} &= \frac{i e_Q g^2 V_{Qq} m_Q m_l \tan^2 \beta}{8 M_W^2 M_{H^+}^2} \bar{q}(p_{\bar{q}})(1 + \gamma_5) \frac{\not{p}_Q - \not{p}_{\gamma} + m_Q}{2 p_Q \cdot p_{\gamma}} \not{f}_{\gamma}^* Q(p_Q) (\bar{l}(1 - \gamma_5) \nu) \\
\mathcal{A}_c^{\text{2HDM}} &= \frac{-i e g^2 V_{Qq} m_Q m_l \tan^2 \beta}{8 M_W^2 M_{H^+}^2} \bar{q}(p_{\bar{q}})(1 + \gamma_5) Q(p_Q) \left(\bar{l} \not{f}_{\gamma}^* \frac{\not{p}_{\gamma} - \not{p}_l + m_l}{2 p_l \cdot p_{\gamma}} (1 - \gamma_5) \nu \right)
\end{aligned} \quad (22)$$

where M_{H^+} is the mass of the charged Higgs boson, and $\tan \beta$ is a free parameter in 2HDM. The leading contribution at order $O((\Lambda_{\text{QCD}}/m_Q)^0)$ is $\mathcal{A}_a^{\text{2HDM}}$, which will vanish. This can be shown immediately, the matrix element can be written as

$$<\gamma|\bar{\nu}_{\bar{q}} \not{f}_{\gamma}^* \frac{\not{p}_{\gamma} - \not{p}_{\bar{q}}}{(p_{\gamma} - p_{\bar{q}})^2 - m_q^2} (1 + \gamma_5) u_Q|P> = <\gamma|\bar{\nu}_{\bar{q}} \not{f}_{\gamma}^* \frac{\not{p}_{\gamma} - \not{p}_{\bar{q}}}{(p_{\gamma} - p_{\bar{q}})^2 - m_q^2} \gamma^\mu (1 - \gamma_5) \frac{p_{Q\mu}}{m_Q} u_Q|P> \quad (23)$$

at the leading order, we can replace $p_{Q\mu}/m_Q$ by $p_{P\mu}/m_P$, and the matrix element can be factorized as [8, 17]

$$<\gamma|\bar{\nu}_{\bar{q}} \not{f}_{\gamma}^* \frac{\not{p}_{\gamma} - \not{p}_{\bar{q}}}{(p_{\gamma} - p_{\bar{q}})^2 - m_q^2} \gamma^\mu (1 - \gamma_5) u_Q|P> = i F_A \epsilon^{\mu\nu\rho\sigma} \varepsilon_{\gamma\nu} p_{\gamma\rho} p_{P\sigma} + F_c (\varepsilon_{\gamma}^\mu p_P \cdot p_\gamma - p_{\gamma}^\mu \varepsilon_{\gamma} \cdot p_P) \quad (24)$$

Contracting with $p_{P\mu}$, it will vanish.

In fact, both Fig. 2. a and Fig. 2. b, and the QCD corrections of them will not contribute to the decay amplitude because of the symmetry of the wave-function of the pseudoscalar mason. Using the similar factorization procedure as in the SM, we find that, up to the order $O(\alpha_s \Lambda_{\text{QCD}}/m_Q)$, the matrix element of Fig. 2. a and Fig. 2. b with 1-loop QCD corrections can be factorized as

$$F_{2\text{HDM}}(\mu) = \sum_n \int d^4 k_Q \int d^4 k_{\bar{q}} \Phi(k_Q, k_{\bar{q}}) C_n^{2\text{HDM}}(k_Q, k_{\bar{q}}, \mu) T_n^{2\text{HDM}}(k_Q, k_{\bar{q}}) \quad (25)$$

the delta-function in the wave-function will replace the transmission momenta k_q, k_Q to the on-shell momenta p_q, p_Q . Using the definition of the wave-function in Eqs. (15) and (16), in Dirac representation, we find

$$F_{2\text{HDM}}(\mu) = \sum_n \int d^3 k \Psi(k) C_n^{2\text{HDM}}(x, \hat{y}, \hat{z}, \hat{w}, \mu) \text{Tr}[M \cdot T_n^{2\text{HDM}}(p_Q, p_{\bar{q}})] \quad (26)$$

with

$$\begin{aligned} \hat{y} &= 2p_Q \cdot p_\gamma, \quad \hat{z} = 2p_{\bar{q}} \cdot p_\gamma, \quad , \hat{y} = 2p_Q \cdot p_{\bar{q}} \\ M &= \frac{(-\frac{1}{2})(\not{p}_Q + m_Q)(1 + \gamma_0)\not{p}_{\bar{q}}\gamma_5}{\sqrt{p_{\bar{q}0}(p_{\bar{q}0} + m_q)p_{Q0}(p_{Q0} + m_Q)}} \end{aligned} \quad (27)$$

and

$$\begin{aligned} T_1^{2\text{HDM}} &= e_q \frac{\not{\epsilon}_\gamma \not{p}_\gamma}{2p_{\bar{q}} \cdot p_\gamma} (1 + \gamma_5), \quad T_2^{2\text{HDM}} = e_q \frac{\not{\epsilon}_\gamma \not{p}_{\bar{q}}}{2p_{\bar{q}} \cdot p_\gamma} (1 + \gamma_5), \quad T_3^{2\text{HDM}} = e_q \frac{\not{\epsilon}_\gamma \not{p}_Q}{2p_{\bar{q}} \cdot p_\gamma} (1 + \gamma_5) \\ T_4^{2\text{HDM}} &= e_Q (1 + \gamma_5) \frac{\not{p}_\gamma}{2p_Q \cdot p_\gamma} \not{\epsilon}_\gamma, \quad T_5^{2\text{HDM}} = e_Q (1 + \gamma_5) \frac{m_Q \not{\epsilon}_\gamma}{2p_Q \cdot p_\gamma} \end{aligned} \quad (28)$$

The kinematic indicates that

$$\begin{aligned} p_\gamma &= (E_\gamma, 0, 0, -E_\gamma), \quad \varepsilon = (0, \varepsilon_1, \varepsilon_2, 0) \\ p_{\bar{q}} &= (p_{\bar{q}0}, k_1, k_2, k_3), \quad p_Q = (p_{Q0}, -k_1, -k_2, -k_3) \end{aligned} \quad (29)$$

Notice that, $C_n^{2\text{HDM}}$ are either even functions of k_1 or k_2 , in this case, after the trace, all terms are vanished, or the odd function of k_1 or k_2 , then, all terms will vanish after the integral $\int d^3 k$.

However, $\mathcal{A}_c^{2\text{HDM}}$ will survive and will contribute to the decay amplitude. As the same case in the SM, $\mathcal{A}_c^{2\text{HDM}}$ also dose not receive contribution from 1-loop hard scattering kernel.

As a result, we find

$$\begin{aligned}\mathcal{A}_{2HDM} &= \frac{eG_f}{\sqrt{2}} \frac{\tan^2 \beta m_l m_Q}{M_{H^+}^2} V_{Qq} < 0 | \bar{q} \Gamma Q | P > \bar{l} \not{f}_\gamma^* \frac{\not{p}_\gamma - \not{p}_l + m_l}{2p_l \cdot p_\gamma} (1 - \gamma_5) \nu \\ &= \frac{eG_f}{\sqrt{2}} \frac{\tan^2 \beta m_l m_Q}{M_{H^+}^2} V_{Qq} f_{2HDM} \bar{l} \not{f}_\gamma^* \frac{\not{p}_\gamma - \not{p}_l + m_l}{2p_l \cdot p_\gamma} (1 - \gamma_5) \nu\end{aligned}\quad (30)$$

with

$$f_{2HDM} = \frac{1}{(2\pi)^3} \frac{3}{\sqrt{6}} \int d^3 k \Psi(k) \frac{1}{2\sqrt{p_{q0} p_{Q0} (p_{q0} + m_q) (p_{Q0} + m_Q)}} (-2m_Q p_{\bar{q}0} - 2p_{\bar{q}} \cdot p_Q) \quad (31)$$

IV. NUMERICAL RESULT AND ANALYSIS OF PARAMETERS OF 2HDM

The form factors defined in Eq. (17) can be calculated using the integral defined in Eq. (18), and we evaluate this integral using [16]

$$\begin{aligned}m_D &= 1.9 \text{ GeV}, \quad m_B = 5.1 \text{ GeV}, \quad m_u = m_d = 0.08 \text{ GeV} \\ m_b &= 4.98 \text{ GeV}, \quad m_c = 1.54 \text{ GeV} \\ \Lambda_{\text{QCD}} &= 200 \text{ MeV}, \quad \lambda_B = 2.8 \text{ GeV}^{-1}, \quad \lambda_D = 3.4 \text{ GeV}^{-1}\end{aligned}\quad (32)$$

The numerical results of the form factors are inconvenient to use when calculate the decay widths. For simplicity, we use some simple forms to fit the numerical results. Inspired by the form factors in Ref. [18], the form factors are fitted as

$$F_{A,V}(E_\gamma) = \left(A_{A,V} \frac{\Lambda_{\text{QCD}}}{E_\gamma} + B_{A,V} \left(\frac{\Lambda_{\text{QCD}}}{E_\gamma} \right)^2 \right) \quad (33)$$

The results are more reliable at the region $E_\gamma \gg \Lambda_{\text{QCD}}$ because we have neglected the higher order terms of $\Lambda_{\text{QCD}}/E_\gamma$. As we have done in Ref. [9], we choose the region $E_\gamma > 2\Lambda_{\text{QCD}}$ to fit the parameters in Eq. (33). The result of B mason in the SM is

$$\begin{aligned}F_A^B(E_\gamma) &= \left(0.25 \frac{\Lambda_{\text{QCD}}}{E_\gamma} + 0.39 \left(\frac{\Lambda_{\text{QCD}}}{E_\gamma} \right)^2 \right) \text{ GeV}^{-1} \\ F_V^B(E_\gamma) &= \left(0.28 \frac{\Lambda_{\text{QCD}}}{E_\gamma} - 0.73 \left(\frac{\Lambda_{\text{QCD}}}{E_\gamma} \right)^2 \right) \text{ GeV}^{-1}\end{aligned}\quad (34)$$

For D mason, there is an additional minus sign in F_A and F_V , and the result is

$$\begin{aligned}F_A^D(E_\gamma) &= \left(-0.10 \frac{\Lambda_{\text{QCD}}}{E_\gamma} + 0.76 \left(\frac{\Lambda_{\text{QCD}}}{E_\gamma} \right)^2 \right) \text{ GeV}^{-1} \\ F_V^D(E_\gamma) &= \left(0.39 \frac{\Lambda_{\text{QCD}}}{E_\gamma} + 0.04 \left(\frac{\Lambda_{\text{QCD}}}{E_\gamma} \right)^2 \right) \text{ GeV}^{-1}\end{aligned}\quad (35)$$

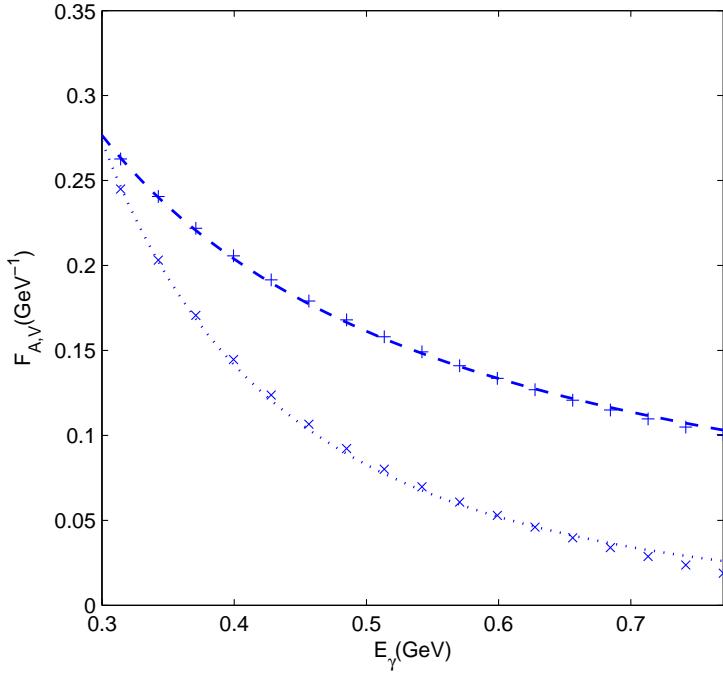


FIG. 3: Fit of the form factors of $B \rightarrow \gamma l \nu_l$. The points ‘ \times ’ and ‘ $+$ ’ are the numerical results for the form factors F_V and F_A , and the dotted and dashed curves are the fitted results using Eq. (33).

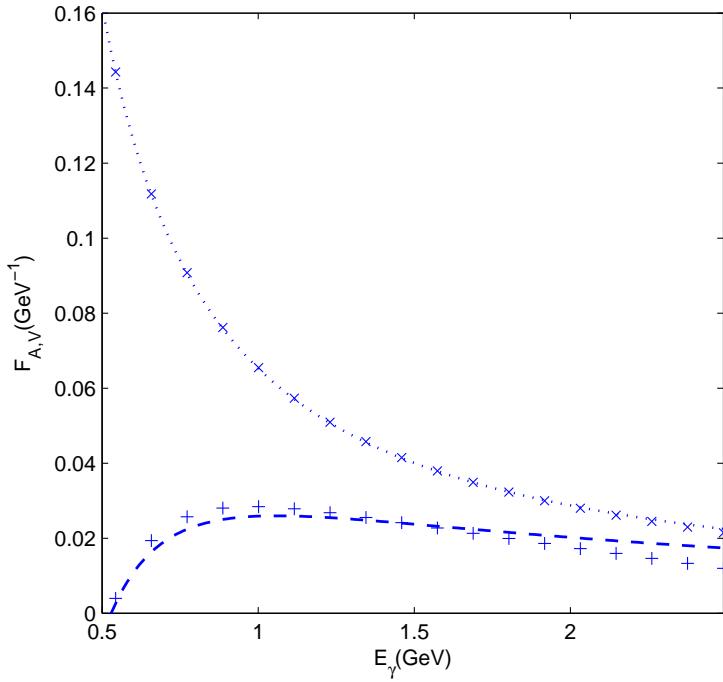


FIG. 4: Fit of the form factors of $D \rightarrow \gamma l \nu_l$. The points ‘ \times ’ and ‘ $+$ ’ are the numerical results for the form factors F_V and F_A , and the dotted and dashed curves are the fitted results using Eq. (33).

The fitting of the form factors are shown in Fig. 3 and Fig. 4.

On the other hand, F_{c1} and F_{c2} are related to the decay constant, and we use the result in Refs. [16, 18] which is calculated using the same wave-function and the same parameters as what we use in this work. The decay constants are

$$f_B = 193.57 \text{ MeV}, \quad f_D = 204.98 \text{ MeV} \quad (36)$$

The contribution of the 2HDM $f_{2\text{HDM}}$ can be calculated using Eq. (31), and the result is

$$f_{2\text{HDM}}^B = -1.23 \text{ GeV}^2, \quad f_{2\text{HDM}}^D = -0.51 \text{ GeV}^2 \quad (37)$$

With R defined as $R = \tan \beta / M_{H^\pm}$, the branch ratios can be written as

$$Br_{\text{tot}} = Br_{\text{SM}} \times (1 + R^2 m_Q m_l a + R^4 m_Q^2 m_l^2 b) \quad (38)$$

with

$$a = \frac{1}{R^2 m_Q m_l} \frac{\mathcal{A}_{\text{SM}}^2}{\mathcal{A}_{\text{SM}}^* \mathcal{A}_{2\text{HDM}} + \mathcal{A}_{2\text{HDM}}^* \mathcal{A}_{\text{SM}}}, \quad b = \frac{1}{R^4 m_Q^2 m_l^2} \frac{\mathcal{A}_{\text{SM}}^2}{\mathcal{A}_{2\text{HDM}}^2} \quad (39)$$

Just as the case of the pure leptonic decay [10], the interference term in the radiative leptonic decay is also found to be destructive. We calculate Br_{SM} , a and b separately. Using the fitted result of F_A , F_V , the result of f_P and $f_{2\text{HDM}}$, and using the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [19, 20], and mass of the leptons

$$V_{cd} = 0.226, \quad V_{ub} = 0.0047, \quad m_\tau = 1776.82 \text{ MeV}, \quad m_\mu = 105.658 \text{ MeV} \quad (40)$$

we can obtain Br_{SM} , a and b . There are IR divergences in the radiative leptonic decays when the photon is soft or collinear with the emitted lepton. Theoretically this IR divergences can be canceled by adding the decay rate of the radiative leptonic decay with the pure leptonic decay rate, in which one-loop correction is included [21]. This is because the radiative leptonic decay can not be distinguished from the pure leptonic decay in experiment when the photon energy is smaller than the experimental resolution to the photon energy. So the decay rate of the radiative leptonic decay depend on the experimental resolution to the photon energy E_γ which is denoted by ΔE_γ . The dependence of the branching ratios of B meson in the SM on the resolution are listed in Table I. And for the same reason, a and b also depend on ΔE_γ , the results of a and b of B meson are listed in Table II.

ΔE_γ	$Br_{\text{SM}}(B \rightarrow \mu\nu_\mu\gamma)$	$Br_{\text{SM}}(B \rightarrow \tau\nu_\tau\gamma)$	ΔE_γ	$Br_{\text{SM}}(B \rightarrow \mu\nu_\mu\gamma)$	$Br_{\text{SM}}(B \rightarrow \tau\nu_\tau\gamma)$
5MeV	1.35×10^{-6}	1.69×10^{-6}	20MeV	1.13×10^{-6}	1.27×10^{-6}
10MeV	1.24×10^{-6}	1.48×10^{-6}	25MeV	1.10×10^{-6}	1.20×10^{-6}
15MeV	1.18×10^{-6}	1.35×10^{-6}	30MeV	1.07×10^{-6}	1.14×10^{-6}

TABLE I: The branching ratios with different photon resolution ΔE_γ in the SM.

ΔE_γ	$a^{B \rightarrow \mu\nu_\mu\gamma}$	$b^{B \rightarrow \mu\nu_\mu\gamma}$	$a^{B \rightarrow \tau\nu_\tau\gamma}$	$b^{B \rightarrow \tau\nu_\tau\gamma}$	ΔE_γ	$a^{B \rightarrow \mu\nu_\mu\gamma}$	$b^{B \rightarrow \mu\nu_\mu\gamma}$	$a^{B \rightarrow \tau\nu_\tau\gamma}$	$b^{B \rightarrow \tau\nu_\tau\gamma}$
5MeV	-7.47	14.11	-2.53	12.49	20MeV	-8.50	10.01	-2.47	10.59
10MeV	-7.98	12.25	-2.51	11.68	25MeV	-8.65	9.19	-2.46	10.15
15MeV	-8.28	10.99	-2.49	11.09	30MeV	-8.77	8.49	-2.45	9.75

TABLE II: a and b defined in Eq. (39) with different photon resolution ΔE_γ .

The result of $D \rightarrow \tau\nu_\tau\gamma$ is too small because of the phase space suppression, so we only calculate the branching ratios of $D \rightarrow \mu\nu_\mu\gamma$, and the result is listed in Table III.

Using $\Delta E_\gamma = 10\text{MeV}$ [22], the Br_{tot} can be written as

$$\begin{aligned} Br_{B \rightarrow \gamma\mu\nu_\mu} &= 1.24 \times 10^{-6} \times r_H = 1.24 \times 10^{-6} \times (1 - 4.20R^2 + 3.39R^4) \\ Br_{B \rightarrow \gamma\tau\nu_\tau} &= 1.48 \times 10^{-6} \times r_H = 1.48 \times 10^{-6} \times (1 - 22.20R^2 + 914.66R^4) \\ Br_{D \rightarrow \gamma\mu\nu_\mu} &= 3.64 \times 10^{-5} \times r_H = 3.64 \times 10^{-5} \times (1 - 1.32R^2 + 0.78R^4) \end{aligned} \quad (41)$$

with r_H defined as $Br_{\text{tot}} / Br_{\text{SM}}$. The numerical results rely on R . In Ref. [10], two allowed regions for R is given as

$$0 < R_1 < 0.15 \text{ GeV}^{-1}, \quad 0.22 \text{ GeV}^{-1} < R_2 < 0.33 \text{ GeV}^{-1} \quad (42)$$

In Ref. [11], the constraint for M_{H^+} is $M_{H^+} > 360 \text{ GeV}$ at 95% CL, and Ref. [12] shows that $M_{H^+} > 315 \text{ GeV}$ at 95% CL.. In Ref. [13], the $\tan\beta$ is also constrained. In the case that $m_h = 126 \text{ GeV}$, $\tan\beta < 5$, while in the case that $m_h < 126 \text{ GeV}$, $M_H = 126 \text{ GeV}$, $\tan\beta < 30$. Considering those constraints, we find $R < 0.1 \text{ GeV}$, and the R_2 region can be excluded.

The branching ratios including the contribution of charged Higgs bosons in the region that $0 < R < 0.15 \text{ GeV}^{-1}$ are listed in Table IV. And the ratio r_H as a function of R are shown in Figs. 5, 6 and 7.

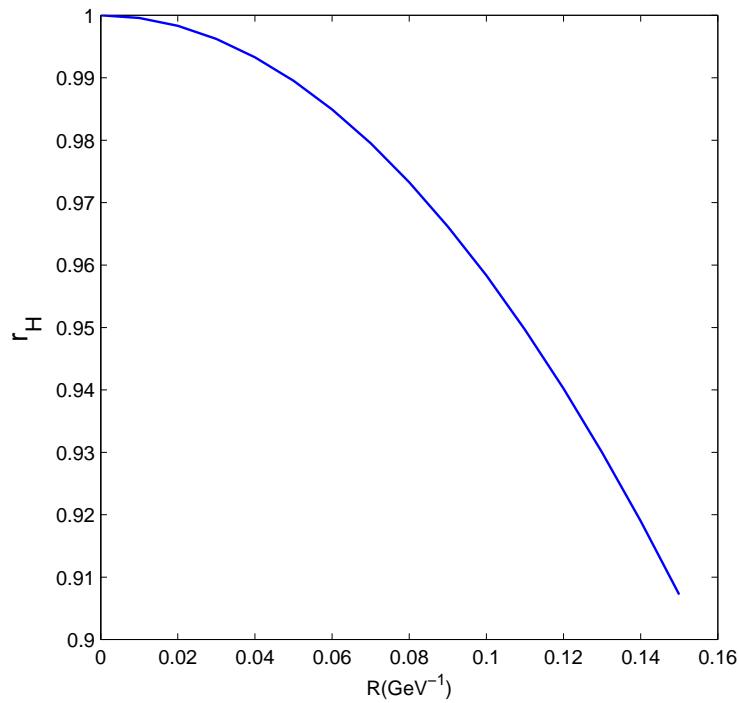


FIG. 5: r_H as function of R in the $B \rightarrow \gamma \mu \nu_\mu$ decay mode.

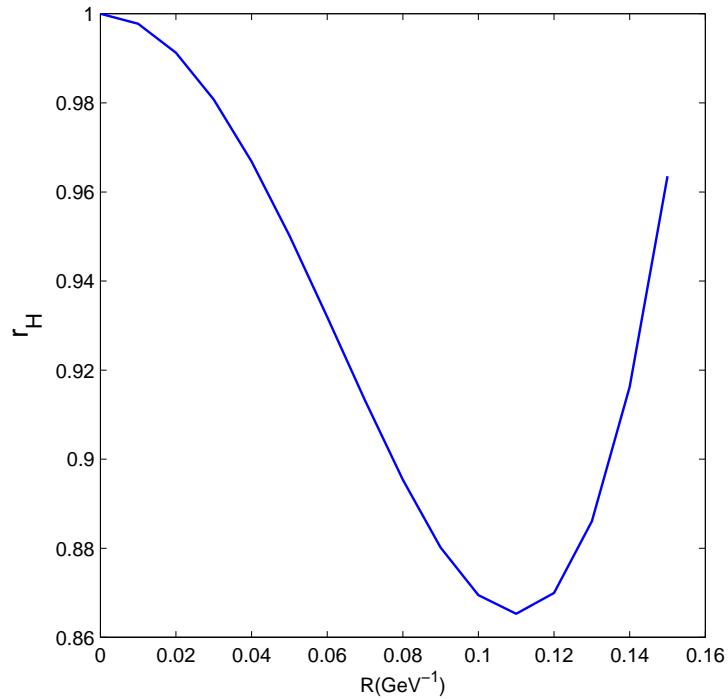


FIG. 6: r_H as function of R in the $B \rightarrow \gamma \tau \nu_\tau$ decay mode.

ΔE_γ	$Br_{\text{SM}}(D \rightarrow \mu\nu_\mu\gamma)$	$a^{D \rightarrow \mu\nu_\mu\gamma}$	$b^{D \rightarrow \mu\nu_\mu\gamma}$	ΔE_γ	$Br_{\text{SM}}(D \rightarrow \mu\nu_\mu\gamma)$	$a^{D \rightarrow \mu\nu_\mu\gamma}$	$b^{D \rightarrow \mu\nu_\mu\gamma}$
5MeV	4.42×10^{-5}	-7.00	31.84	20MeV	2.87×10^{-5}	-9.47	25.69
10MeV	3.64×10^{-5}	-8.09	29.46	25MeV	2.62×10^{-5}	-9.97	23.98
15MeV	3.18×10^{-5}	-8.86	27.49	30MeV	2.42×10^{-5}	-10.38	22.31

TABLE III: The branching ratios with different photon resolution ΔE_γ of $D \rightarrow \gamma\mu\nu_\mu$ in the SM, and the contribution of 2HDM indicated by a and b defined in Eq. (39).

	Br in $0 < R < 0.15 \text{ GeV}^{-1}$
$B \rightarrow \gamma\mu\nu_\mu$	$1.12 \times 10^{-6} < Br_{\text{tot}} < 1.24 \times 10^{-6}$
$B \rightarrow \gamma\tau\nu_\tau$	$1.28 \times 10^{-6} < Br_{\text{tot}} < 1.48 \times 10^{-6}$
$D \rightarrow \gamma\mu\nu_\mu$	$3.53 \times 10^{-5} < Br_{\text{tot}} < 3.64 \times 10^{-5}$

TABLE IV: The branching ratios in the region $0 < R < 0.15 \text{ GeV}^{-1}$

We find that, the decay mode $B \rightarrow \gamma\tau\nu_\tau$ is very sensitive to the contribution of the charged Higgs bosons. For $R = 0.1 \text{ GeV}^{-1}$, for example, the branching ratios is suppressed by more than 13%.

V. SUMMARY

In this paper, we calculated the branching ratios of the radiative leptonic decay of the heavy pseudoscalar meson with a massive lepton. The contribution of 2HDM-Type-II is included. The SM contribution is obtained using the factorization procedure up to the order $O(\alpha_s \Lambda_{\text{QCD}} / m_Q)$ with one-loop correction. The contribution of the charged Higgs boson is also obtained by the factorization scheme, however, we find that only the diagram with the photon emitting from the lepton leg will contribute, which is an order $O(\Lambda_{\text{QCD}} / m_Q)$ contribution and dose not receive contributions from 1-loop hard scattering kernel. The numerical results of the branching ratios are listed in Table IV, and dependence of r_H on R is shown in Figs. 5, 6 and 7.

We find that, the decay mode $B \rightarrow \gamma\tau\nu_\tau$ is sensitive to the contribution of the charged Higgs in the 2HDM. This decay mode is as sensitive as the pure leptonic decay of B meson, which is estimated to be $r_H = (1 - m_P^2 R^2)$ [10]. However, the result of the pure leptonic decay is derived at the leading order of $O((\Lambda_{\text{QCD}} / m_Q)^0)$, while our result is calculated up to

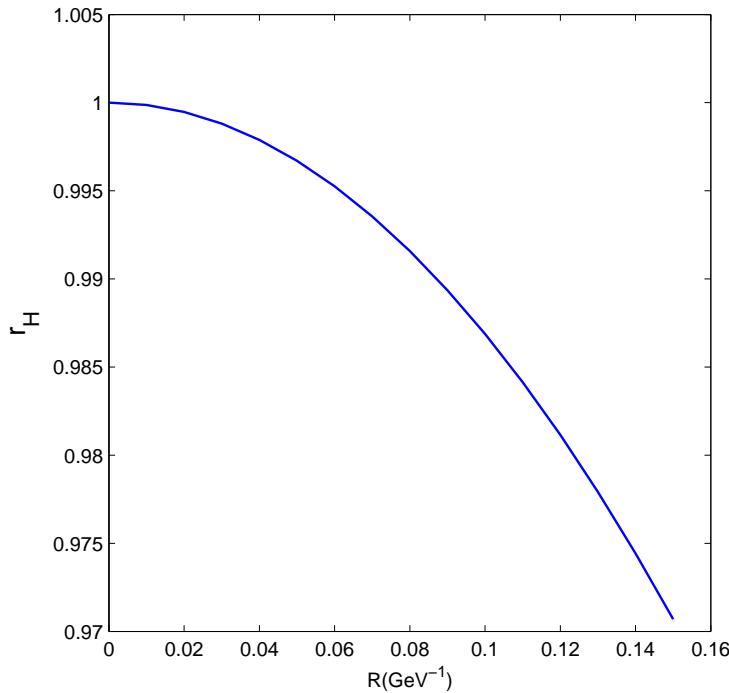


FIG. 7: r_H as function of R in the $D \rightarrow \gamma\mu\nu_\mu$ decay mode.

the order $O(\alpha_s \Lambda_{\text{QCD}}/m_Q)$. On the other hand, the branching ratios of this decay mode in the SM is about 1.48×10^{-6} , which is larger than $B \rightarrow e\nu_e$ and $B \rightarrow \mu\nu_\mu$ both are believed to be less than 10^{-6} [19]. The decay mode $B \rightarrow \gamma\mu\nu_\mu$ is also important because the charged Higgs can suppress the branching ratios as large as about 10%. It is more sensitive than the pure leptonic decay modes $D \rightarrow l\nu$, which is estimated to be $r_H = (1 - m_P^2 R^2) = (1 - 3.6R^2)$. Though the branching ratio of the radiative leptonic decay of the B meson is much smaller, the result in the SM is more reliable than the case of the D meson because the heavy quark mass m_b is larger than m_c .

We also find that, the decay mode $D \rightarrow \mu\nu_\mu\gamma$ is rarely affected by the charged Higgs boson. On the other hand, the numerical result of this mode is also not sufficiently accurate in the SM because the mass of c quark is not so heavy. This decay mode should be excluded in the search of the charged Higgs bosons.

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